

830017

摘要:

Bayes, Pareto, 8, yes, U, Bayes, 8, 3P ayeto

1 预备知识

定义 1^[4]

$$\begin{aligned} & () \quad () \\ & () \quad () \end{aligned}$$

引理 1^[10]

证明 ()
()

$$() \frac{ () }{ () }$$

$$(1) \quad () \quad ()$$

$$(1) \frac{ () (1) }{ () (1) } \frac{ () }{ () } \quad ()$$

$$() \quad \left| \quad () \right.$$

$$() -$$

$$() \frac{ () }{ () }$$

2 形状参数的极大似然估计和Bayes估计

2.1 形状参数的极大似然估计

$$(1) \quad () \left[\quad \right]$$

$$\begin{aligned} & \left(\frac{\Gamma(n)}{\Gamma(n-1)} \right) \left(\frac{\Gamma(n-1)}{\Gamma(n-2)} \right) \left(\frac{\Gamma(n-2)}{\Gamma(n-3)} \right) \cdots \left(\frac{\Gamma(2)}{\Gamma(1)} \right) \\ & \quad \left(\frac{1}{x} \right)^{n-1} \left[\frac{\Gamma(n)}{\Gamma(n-1)} \right] \left(- \right) \end{aligned}$$

$$\frac{1}{x} \left(- \right)$$

2.2 二次损失函数下形状参数的 Bayes 估计

定义 2^[15]

$$\left(\frac{\Gamma(n)}{\Gamma(n-1)} \right) \left(\frac{\Gamma(n-1)}{\Gamma(n-2)} \right)$$

引理 2^[15]

$$\frac{\left(\frac{\Gamma(n)}{\Gamma(n-1)} \right)}{\left(\frac{\Gamma(n-1)}{\Gamma(n-2)} \right)}$$

定理 1

$$\left(\frac{\Gamma(n)}{\Gamma(n-1)} \right) \left(\frac{\Gamma(n-1)}{\Gamma(n-2)} \right)$$

$$\begin{aligned}
 & \frac{\binom{r-1}{k} \binom{r-1}{r-k}}{\binom{r-1}{k} \binom{r-1}{r-k}} \frac{\binom{r-1}{k}}{\binom{r-1}{k}} \binom{r-1}{r-k} \\
 & \frac{\binom{r-1}{k}}{\binom{r-1}{k}} \binom{r-1}{r-k} \frac{\binom{r-1}{k}}{\binom{r-1}{k}} \binom{r-1}{r-k} \\
 & \frac{\binom{r-1}{k}}{\binom{r-1}{k}} \binom{r-1}{r-k} \frac{\binom{r-1}{k}}{\binom{r-1}{k}} \binom{r-1}{r-k} \\
 & \frac{\binom{r-1}{k}}{\binom{r-1}{k}} \frac{\binom{r-1}{r-k}}{\binom{r-1}{r-k}}
 \end{aligned}$$

2.3 Q对称熵损失函数下形状参数的Bayes估计

定义3^[16]

$$\binom{r-1}{k} \binom{r-1}{r-k} \binom{r-1}{k}$$

引理3^[16]

$$\left[\frac{\binom{r-1}{k}}{\binom{r-1}{k}} \right]^{-1}$$

定理3

$$\binom{r-1}{k} \binom{r-1}{r-k}$$

$$\left[\frac{\binom{r-1}{k}}{\binom{r-1}{k}} \right]^{-1}$$

证明

$$\binom{r-1}{k} \frac{\binom{r-1}{r-k}}{\binom{r-1}{r-k}}$$

$$\begin{aligned}
 & \binom{r-1}{k} \binom{r-1}{r-k} \frac{\binom{r-1}{k}}{\binom{r-1}{k}} \\
 & \frac{\binom{r-1}{k}}{\binom{r-1}{k}} \frac{\binom{r-1}{r-k}}{\binom{r-1}{r-k}}
 \end{aligned}$$

$$\begin{aligned}
 & \binom{r-1}{k} \binom{r-1}{r-k} \frac{\binom{r-1}{k}}{\binom{r-1}{k}} \\
 & \frac{\binom{r-1}{k}}{\binom{r-1}{k}} \frac{\binom{r-1}{r-k}}{\binom{r-1}{r-k}}
 \end{aligned}$$

$$\left[\frac{\binom{r-1}{k}}{\binom{r-1}{k}} \right]^{-1} \left[\frac{\binom{r-1}{k}}{\binom{r-1}{k}} \right]^{-1}$$

定理4

$$\binom{r-1}{k} \binom{r-1}{r-k}$$

$$\left[\frac{\binom{r-1}{k}}{\binom{r-1}{k}} \right]^{-1}$$

证明

$$\begin{aligned}
 & \binom{r-1}{k} \frac{\binom{r-1}{k}}{\binom{r-1}{k}} \binom{r-1}{k} \\
 & \binom{r-1}{k} \binom{r-1}{k} \frac{\binom{r-1}{k}}{\binom{r-1}{k}} \binom{r-1}{k} \\
 & \frac{\binom{r-1}{k}}{\binom{r-1}{k}} \binom{r-1}{k} \frac{\binom{r-1}{k}}{\binom{r-1}{k}} \binom{r-1}{k} \\
 & \binom{r-1}{k} \binom{r-1}{k} \frac{\binom{r-1}{k}}{\binom{r-1}{k}} \binom{r-1}{k} \\
 & \frac{\binom{r-1}{k}}{\binom{r-1}{k}} \binom{r-1}{k} \frac{\binom{r-1}{k}}{\binom{r-1}{k}} \binom{r-1}{k}
 \end{aligned}$$

$$\left[\frac{\binom{r-1}{k}}{\binom{r-1}{k}} \right]^{-1} \left[\frac{\binom{r-1}{k}}{\binom{r-1}{k}} \right]^{-1} \dots$$

3 形状参数的多层 Bayes 估计和 E-Bayes 估计

3.1 形状参数的多层 Bayes 估计

$$\begin{aligned}
 & \binom{r-1}{k} \\
 & \binom{r-1}{k} \binom{r-1}{k} \\
 & \binom{r-1}{k} \binom{r-1}{k} -
 \end{aligned}$$

定理 5

$$\begin{aligned}
 & \binom{r-1}{k} \\
 & \binom{r-1}{k}
 \end{aligned}$$

$$\frac{\frac{\binom{r-1}{k}}{\binom{r-1}{k}}}{\frac{\binom{r-1}{k}}{\binom{r-1}{k}}}$$

证明

$$\binom{r-1}{k} \binom{r-1}{k} \binom{r-1}{k} - \frac{\binom{r-1}{k}}{\binom{r-1}{k}}$$

$$\binom{r-1}{k} \frac{\binom{r-1}{k} \binom{r-1}{k}}{\binom{r-1}{k} \binom{r-1}{k}} = \frac{\frac{\binom{r-1}{k}}{\binom{r-1}{k}} \binom{r-1}{k}}{\frac{\binom{r-1}{k}}{\binom{r-1}{k}}}$$

$$\binom{r-1}{k} \binom{r-1}{k} \frac{\frac{\binom{r-1}{k}}{\binom{r-1}{k}} \binom{r-1}{k}}{\frac{\binom{r-1}{k}}{\binom{r-1}{k}}} = \frac{\frac{\binom{r-1}{k}}{\binom{r-1}{k}} \binom{r-1}{k}}{\frac{\binom{r-1}{k}}{\binom{r-1}{k}}}$$

$$\begin{aligned}
 & \frac{\binom{r}{k} \binom{r-k}{l}}{\binom{r}{k+l}} = \frac{\binom{r}{k} \binom{r-k}{l}}{\binom{r}{k+l}} \\
 & \frac{\binom{r}{k} \binom{r-k}{l}}{\binom{r}{k+l}} = \frac{\binom{r}{k} \binom{r-k}{l}}{\binom{r}{k+l}} \\
 & \frac{\binom{r}{k} \binom{r-k}{l}}{\binom{r}{k+l}} = \frac{\binom{r}{k} \binom{r-k}{l}}{\binom{r}{k+l}}
 \end{aligned}$$

3.2 形状参数的E-Bayes估计

定义4^[17] 设 (X, Y) 服从二元分布 (π, ρ)

其联合分布函数为 $F(x, y)$ ，边缘分布函数分别为 $F_X(x)$ 和 $F_Y(y)$

定理6 设 (X, Y) 服从二元分布 (π, ρ)

$$F(x, y) = \int_0^x \int_0^y f(t, s) dt ds$$

证明

$$F(x, y) = \int_0^x \int_0^y f(t, s) dt ds$$

$$\begin{aligned}
 & \frac{\partial F(x, y)}{\partial x} = \int_0^y f(x, s) ds \\
 & \frac{\partial F(x, y)}{\partial y} = \int_0^x f(t, y) dt
 \end{aligned}$$

表1 n, m 取不同值时, $\theta_M, \delta_{11}, \delta_{12}$ 的模拟结果

表2 n, m 取不同值时, δ_{21}, δ_{22} 的模拟结果

表3 n, m 取不同值时, H_B, E_B 的模拟结果

5 实例分析

