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[12] [13]

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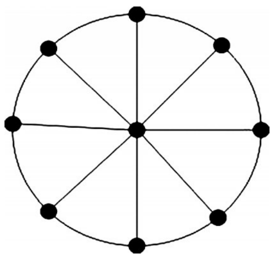
1 [14] G H $V(G\square H)=V(G)\times V(H)$ (u_1, v_1)
(u_2,v_2) $u_1=u_2$ v_1v_2 $E(H)$ $v_1=v_2$ u_1u_2 $E(G)$ 1

2 [15] G H $V(G\times H)=V(G)\times V(H)$

7 $N_{a,b}$ $b + 1$ P_{b+1} a C_a 6
8 a C_a b C_b $b + 1$ $P_{b+1}(b \geq 1)$ C_a
C_b P_{b+1} $D_{a,b,c}$ 7



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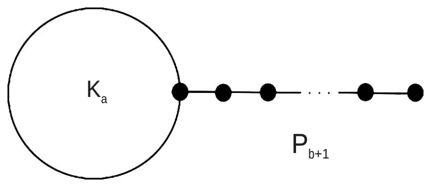

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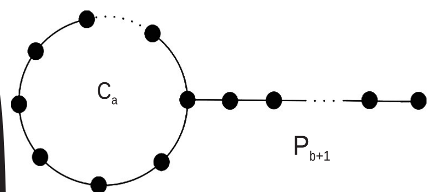

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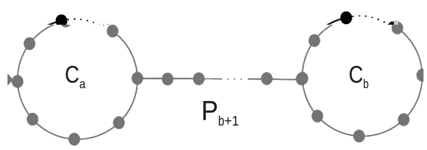

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1 Sombor

Sombor

1 $G = P_n \square W_m$ Sombor

$$SO(G) = (11mn - 9n - 20m + 14)\sqrt{2} + 2\sqrt{2m^2 + 2m + 1} + (n - 2)(m - 1)\sqrt{m^2 + 2m + 26} + 2(m - 1)\sqrt{41} + 2(m - 1)\sqrt{m^2 + 16}$$

$v(G) = mn$ $e(G)$ $\delta_G = 4$ $\Delta_G = m + 1$ G 2
m $(n - 2)(m - 1)$

$$= 2\sqrt{(m+1)^2 + m^2} + (n-2)(m-1)\sqrt{(m+1)^2 + 5^2} + (2n-5)(m-1)\sqrt{5^2+5^2}$$

$$+2(m-1)\sqrt{4^2+5^2} + 2(m-1)\sqrt{4^2+4^2} + 2(m-1)\sqrt{4^2+m^2} + (n-3)\sqrt{(m+1)^2+(m+1)^2}$$

$$= 2\sqrt{2m^2+2m+1} + (n-2)(m-1)\sqrt{m^2+2m+26} + 5(2n-5)(m-1)\sqrt{2} + 2(m-1)\sqrt{41}$$

$$+8(m-1)\sqrt{2} + 2(m-1)\sqrt{16+m^2} + (n-3)(m+1)\sqrt{2}$$

$$= (11mn - 9n - 20m + 14)\sqrt{2} + 2\sqrt{2m^2+2m+1} + (n-2)(m-1)\sqrt{m^2+2m+26}$$

$$+2(m-1)\sqrt{41} + 2(m-1)\sqrt{m^2+16}$$

2 $G = P_n \square F_m$ Sombor

$$SO(G) = (11mn - 26n - 20m + 26)\sqrt{2} + 2\sqrt{2m^2+2m+1} + 2(n-2)\sqrt{m^2+2m+17} + 4\sqrt{m^2+9}$$

$$+2(m+n-5)\sqrt{41} + 2(m-3)\sqrt{16+m^2} + (mn-3n-2m+6)\sqrt{m^2+2m+26} + 40$$

$v(G) = mn$ $e(G) = 3mn - m - 3n.$ G $\delta_G = 3$ ${}_G = m + 1$ G 2

m $2(n-2)$ 4 $(n-3)$ 8

4 4 m $2(m+n-7)$ 4 $2(m+n-5)$ 4 5

$(2mn - 5m - 7n + 17)$ 5 $2(m-3)$ m 4 $(mn - 3n - 2m + 6)$

5 G Sombor

3 $G = W_n \square W_m$ Sombor

$$SO(G) = (n-1)\sqrt{(n+m-2)^2 + (m+2)^2} + (m-1)\sqrt{(n+m-2)^2 + (n+2)^2}$$

$$+(n-1)(m-1)\left(\sqrt{m^2+4m+40} + \sqrt{n^2+4n+40}\right) + (14mn - 11n - 11m + 8)\sqrt{2}$$

$v(G) = mn$ $e(G) = 4mn - 2n - 2m.$ G $\delta_G = 6$ ${}_G = m + n - 2$

$(n-1)$ $(m+2)$ $(m-1)$ $(n+2)$ $(n-1)(m-1)$

$(m+2)$ $(n-1)(m-1)$ $(n+2)$ $2(n-1)(m-1)$

$(n-1)$ $(m+2)$ $(m-1)$ $(n+2)$ G

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4 $G = W_n \square F_m$ Sombor

$$SO(G) = 2\left(\sqrt{(m+n-2)^2 + (n+1)^2} + \sqrt{(n+1)^2 + (n+2)^2}\right) + (m-3)\sqrt{(m+n-2)^2 + (n+2)^2}$$

$$+(n-1)\sqrt{(m+n-2)^2 + (m+2)^2} + (14mn - 34n - 11m + 22)\sqrt{2} + 2(n-1)$$

$$\left(\sqrt{n^2+2n+26} + \sqrt{m^2+4m+29} + \sqrt{61}\right) + (n-1)(m-3)\left(\sqrt{m^2+4m+40} + \sqrt{n^2+4n+40}\right)$$

$v(G) = mn$ $e(G) = 4mn - 3n - 2m.$ G $\delta_G = 5$ ${}_G = m + n - 2$ 2

$(n+1)$ $(m-3)$ $(n+2)$ $(n-1)$

$(m+2)$ $(2n-2)$ $(2n-2)$ 6 $(2n-2)$

$(n+1)$ $(2n-2)$ $(m+2)$ $(n-1)(m-3)$ $(m+2)$ 6

$(n-1)(m-3)$ $(n+2)$ 6 $(2mn - 2m - 7n + 7)$ 6 2 $(n+1)$

$(n+2)$ $(n-1)$ $(m+2)$ $(m-4)$ $(n+2)$

$$SO(G)=\frac{1}{2}n\left(n^2a+n^2b+n^2c-n^2+2na+2nb+2nc-10n+a+b+c-13\right)\sqrt{2}+6n\sqrt{2n^2+6n+5}$$

$v(G)=n(a+b+c-1)$ $e(G)=\frac{1}{2}n(na+nb+nc-n+a+b+c+1)$. G $\delta_G=n+1$

${}_G=n+2$ $n(n-1)$ $6n$

$\frac{1}{2}n(na+a+nb+b+nc+c-3n-9)$ G Sombor

$$SO(G)=\frac{1}{2}n\left(n^2a+n^2b+n^2c-n^2+2na+2nb+2nc-10n+a+b+c-13\right)\sqrt{2}+6n\sqrt{2n^2+6n+5}$$

8 $G=N_{a,b}\square K_n$ Sombor

$$SO(G)=\frac{1}{2}n\left(an^2+bn^2+2an+2bn+a+b-8n-8\right)\sqrt{2}+3n\sqrt{2n^2+6n+5}+n\sqrt{2n^2+2n+1}$$

$v(G)=n(a+b)$ $e(G)=\frac{1}{2}n(na+nb+a+b)$ G $\delta_G=n$ ${}_G=n+2$

$\frac{1}{2}n(n-1)$ $3n$ (δ_G+1) $\frac{1}{2}n(an+bn-2n+a+b-6)$

(δ_G+1) n (δ_G+1) $\frac{1}{2}n(n-1)$ G Sombor

$$SO(G)=\frac{1}{2}n\left(an^2+bn^2+2an+2bn+a+b-8n-8\right)\sqrt{2}+3n\sqrt{2n^2+6n+5}+n\sqrt{2n^2+2n+1}$$

3 Sombor

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9 G n k- H

$$SO(G\times H)=nk^2SO(H)$$

$V(G)=\{u_1,u_2,\quad,u_n\}$ $V(H)=\{v_1,v_2,\quad,v_m\}$ $E(G)=\{u_iu_j|1\le i,j\le n\}$ $E(H)=\{v_sv_l|1\le s,l\le m\}$

$V(G\times H)=\{u_iv_s|1\le i\le n,1\le s\le m\}$ $E(G\times H)=\{(u_iv_s)(u_jv_l)|1\le i,j\le n,1\le s,l\le m\}$ $d_G(u_i)=k$

$d_{G\times H}(u_iv_s)=d_G(u_i)d_H(v_s)=kd_H(v_s)$. $e_1=(u_iv_s)(u_jv_l)$ $E(G\times H)$ $e_2=(u_jv_s)(u_iv_l)$ $E(G\times H)$

$e(G\times H)=2e(G)e(H)$ $d_{G\times H}(u_iv_s)=d_{G\times H}(u_jv_s)$ $d_{G\times H}(u_iv_l)=d_{G\times H}(u_jv_l)$

$$SO(G\times H)=\sum_{(u_iv_s)(u_jv_l)\quad G\times H}\sqrt{d_{G\times H}{}^2(u_iv_s)+d_{G\times H}{}^2(u_jv_l)}=2k\frac{nk}{2}\sum_{v_sv_l\quad H}\sqrt{d_H{}^2(v_s)+d_H{}^2(v_l)}=nk^2SO(H)$$

4 Sombor

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10 1 $SO(K_n\square P_m)=2n\sqrt{2n^2+2n+1}+\left(\frac{1}{2}mn^3+mn^2+\frac{1}{2}mn-4n^2-2n\right)\sqrt{2}$

2 $SO(K_n\square C_m)=\frac{1}{2}nm(m+1)(n+1)\sqrt{2}$

3 $SO(K_n\times P_m)=(2n^3-4n^2+2n)\sqrt{5}+(2n^3m-4n^2m-6n^3+12n^2+2nm-6n)\sqrt{2}$ $(m>2)$

4 $SO(K_n\times C_m)=nm(m-2)(n-1)\sqrt{2}$

5 $SO(K_n\quad P_m)=2n^2\sqrt{13n^2-10n+2}+\left(\frac{9}{2}n^3m-3n^2m-10n^3+4n^2+\frac{1}{2}nm\right)\sqrt{2}$ $(m>2)$

6 $SO(K_n \quad C_m) = \frac{1}{2}nm(3n-1)^2\sqrt{2}$

7 $SO(P_n \times P_m) = 4\sqrt{17} + (8m+8n-48)\sqrt{5} + (8mn-24m-24n+80)\sqrt{2}\ (m,n>2)$

8 $SO(C_n \times C_m) = 8mn\sqrt{2}$

9 $SO(C_n \times P_m) = 8n\sqrt{5} + (8nm-12n)\sqrt{2}\ (m>2)$

10 $SO(P_n \quad P_m) = (32mn-78m-78n+200)\sqrt{2} + 8\sqrt{34} + 4\sqrt{53} + (6m+6n-32)\sqrt{89}\ (m,n>2)$

11 $SO(C_n \quad C_m) = 32mn\sqrt{2}$

12 $SO(C_n \quad P_m) = (32mn-78n)\sqrt{2} + 6n\sqrt{89}$.

1 $v(K_n \square P_m) = mn$ $e(K_n \square P_m) = \frac{1}{2}mn^2 + \frac{1}{2}mn - n$ $K_n \square P_m$ $\delta_{K_n \square P_m} = n$ $_{K_n \square P_m} = n+1$ $n^2 - n$ $2n$ $\left(\frac{1}{2}mn^2 + \frac{1}{2}mn - n^2 - 2n\right)$ $K_n \square P_m$ Sombor

2 [7] G n k - G Sombor $SO(G) = \frac{nk^2}{\sqrt{2}}$ $K_n \square C_m$ mn $(n+1)$ - $SO(K_n \square C_m) = \frac{1}{2}nm(m+1)(n+1)\sqrt{2}$

3 $K_n \times P_m$ 10 K_n n $n-1$-

$$SO(K_n \times P_m) = nk^2 SO(H) = n(n-1)^2\left[2\sqrt{5} + 2(m-3)\sqrt{2}\right]$$
$$= (2n^3 - 4n^2 + 2n)\sqrt{5} + (2n^3m - 4n^2m - 6n^3 + 12n^2 + 2nm - 6n)\sqrt{2}\ (m>2)$$

4 $K_n \times C_m$ mn $(2n-2)$ - $SO(K_n \times C_m) = nm(m-2)(n-1)\sqrt{2}$

5 $v(K_n \quad P_m) = mn$ $e(K_n \quad P_m) = \frac{3}{2}mn^2 - n^2 - \frac{1}{2}mn$ $K_n \quad P_m$ $\delta_{K_n \quad P_m} = 2n-1$ $_{K_n \quad P_m} = 3n-1$ $n^2 - n$ $2n^2$ $\left(\frac{3}{2}mn^2 - 4n^2 - \frac{1}{2}mn + n\right)$ $K_n \quad P_m$ Sombor

6 $K_n \quad C_m$ mn $(3n-1)$ -

$$SO(K_n \quad C_m) = \frac{1}{2}nm(3n-1)^2\sqrt{2}$$

7 $v(P_n \times P_m) = mn$ $e(P_n \times P_m) = 2(m-1)(n-1)$ $P_n \times P_m$ $\delta_{P_n \times P_m} = 1$ $_{P_n \times P_m} = 4$ 4 4 2 $(4m+4n-24)$ 2 $(2mn-6m-6n+18)$ $P_n \times P_m$ Sombor

8 $C_n \times C_m$ 10 C_n 2- $SO(C_n \times C_m) = nk^2 SO(C_m) = n2^2 2m\sqrt{2} = 8mn\sqrt{2}$

9 $C_n \times P_m$ 10 C_n 2- $SO(C_n \times P_m) = nk^2 SO(P_m) = n2^2\left[2\sqrt{5} + 2(m-3)\sqrt{2}\right] = 8n\sqrt{5} + (8nm-12n)\sqrt{2}$

10 $v(P_n \quad P_m) = mn$ $e(P_n \quad P_m) = 4mn - 3m - 3n + 2$ $P_n \quad P_m$ $\delta_{P_n \quad P_m} = 3$ $_{P_n \quad P_m} = 8$ 8 5 4 $(2m+2n-8)$ 5 $(6m+6n-32)$ 5 $(4mn-11m-11n+30)$ $P_n \quad P_m$ Sombor

11 $C_n \quad C_m$ mn 8- $SO(C_n \quad C_m) = \frac{mn8^2}{\sqrt{2}} = 32mn\sqrt{2}$

12 $v(C_n \quad P_m) = mn$ $e(C_n \quad P_m) = 4mn - 3n$ $C_n \quad P_m$ $\delta_{C_n \quad P_m} = 5$ $_{C_n \quad P_m} = 8$

$2n$ $6n$ $(4mn - 11n)$

$C_n \quad P_m$ Sombor

11 G $G = G_1 \quad G_2 \quad G_3 \quad G_n$

$$SO(G) > \sum_{i=1}^{n} SO(G_i)$$

$G_1, G_2, G_3, \quad , G_n$ G $G = G_1 \quad G_2 \quad G_3 \quad G_n$ $G_i (1 \le i \le n)$ $E(G_i) \neq \Phi$ $E(G) = E(G_1) + E(G_2) + E(G_3) + \quad + E(G_n)$ $G_i (1 \le i \le n)$ $v \quad V(G_i)$ $v \quad V(G)$ $d_G(v) \ge d_{G_i}(v)$ $e = uv \quad E(G_i)$ $e \quad E(G)$ G_i $d_{G_i}(u)$ G $d_G(u)$ $d_G(u) > d_{G_i}(u)$ $\sqrt{d_G{}^2(v) + d_G{}^2(u)} > \sqrt{d_{G_i}{}^2(v) + d_{G_i}{}^2(u)}$ $E(G_i) \quad E(G)$ $SO(G) > SO(G_i)$ $E(G) = E(G_1) + E(G_2) + E(G_3) + \quad + E(G_n)$ $SO(G) > \sum_{i=1}^{n} SO(G_i)$.

1 G H

$$SO(G \quad H) > SO(G \times H) + SO(G \square H)$$

$G \quad H = (G \times H) \quad (G \square H)$ $E(G \quad H) = E(\quad)$

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Abstract Sombor index is a new topological index based on vertex degree introduced by Gutman in Chemical Graph Theory. In this paper Sombor indices of cartesian products of path P_n with fan graph F_m and wheel graph W_m wheel graph W_n with fan graph F_m and wheel graph W_m fan graph F_n with fan graph F_m and lollipop graph $N_{a,b}$ barbell graph $D_{a,b,c}$ and kite graph $L_{a,b}$ with complete graph K_n are discussed Sombor indices of Direct products Cartesian products and Strong products of complete graph path and cycleare are also studied and the exact index values and some relations of sombor indices about product graph are obtained.

Keywords Sombor index Cartesian product Direct product Strong product

9

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Abstract Based on gradually increasing type truncated samples. Firstly obtain the maximum likelihood estimation of the Pareto distribution shape parameter considering the two loss functions and the two prior distributions of shape parameters four Bayes estimation of the distribution shape parameter is concluded. It is found from the numerical simulation results that the mean square error of the four Bayes estimates is less than the maximum likelihood estimate. Among them when the loss function is a quadratic loss function and the prior distribution of the shape parameter is a conjugate prior distr rmmz H m