图变换及其在图的最小特征值的应用

*

G $V(G)=\{v_1,v_2,\cdots,v_n\}$. G $A(G)=(a_{ij})_{n\times n}$ v_i v_j

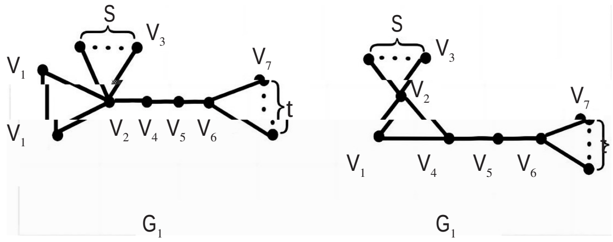
G $a_{ij}=1$ $a_{ij}=0$. $A(G)$ $\lambda_1(G)\geqslant\lambda_2(G)\geqslant\cdots\geqslant\lambda_n(G)$ $A(G)$

G $n\geqslant 12$

9

$C_n^c(n \geq 12)$ $K_{1,n-1}+e$

$C_n^c(n \geq 12)$



G_1 G_1

G G

$$\sum_{v_iv_j\in E(G)} X_{v_i}X_{v_j} \leqslant \sum_{v_iv_j\in E(F)} X_{v_i}X_{v_j} \quad 3$$

$F\in\{G_1,G_2\}$ 1 F uw $u\in V_F^+$ $w\in V_F^-$. F

T_2 $\bar{F}$ F

$$\sum_{v_iv_j\in E(F)} X_{v_i}X_{v_j} \leqslant \sum_{v_iv_j\in E(\bar{F})} X_{v_i}X_{v_j} \quad 4$$

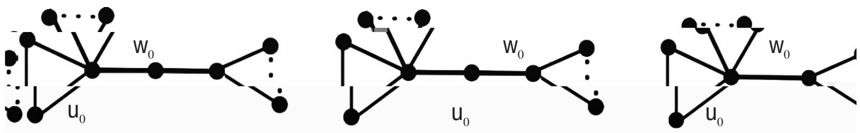
$\bar{F}$ u'' w'' $V_{\bar{F}}^+$ $V_{\bar{F}}^-$ $X_{u''}\geqslant X_v$ $V_{\bar{F}}^+$ $X_{w''}\leqslant X_v$

$V_{\bar{F}}^-$. $\bar{F}$ T_3 $\tilde{F}$ u_0w_0.

$$\sum_{v_iv_j\in E(\bar{F})} X_{v_i}X_{v_j} \leqslant \sum_{v_iv_j\in E(\tilde{F})} X_{v_i}X_{v_j} \quad 5$$

$\tilde{F}\notin\{G_1,G_2\}$ $X_{\hat{u}}$ $X_{\hat{w}}$ $\tilde{F}_+$ $\tilde{F}_-$ $\tilde{F}_+$ $\tilde{F}_-$ T_1 -

$\tilde{F}_+$ $\tilde{F}_-$ $K_{1,p+e}$ $K_{1,q}$. $\tilde{F}$ 2 G_3 G_4 G_5



G_3 G_4 G_5

G G G

$G_s(s=3,4,5)$

$$\sum_{v_iv_j\in E(\tilde{F})} X_{v_i}X_{v_j} \leqslant \sum_{v_iv_j\in E(G_s)} X_{v_i}X_{v_j} \quad 6$$

$\tilde{u}\in V_{G_s}^+$ $\tilde{w}\in V_{G_s}^-$ $X_{\tilde{u}}\leqslant X_v$ $v\in V_{G_s}^+$ $X_{\tilde{w}}\geqslant X_v$ $v\in V_{G_s}^-$. u_0w_0 $\tilde{u}\tilde{w}$

G_* G_1 G_2

$$\sum_{v_iv_j\in E(G_s)} X_{v_i}X_{v_j} \leqslant \sum_{v_iv_j\in E(G_*)} X_{v_i}X_{v_j} \quad 7$$

2 C_l

$v\in C_l$ C_l v' v'' $X_vX_{v'}\leqslant 0$ $X_vX_{v''}\leqslant 0$ C_l T_2 –

C_l $C_{l'}$ $v\in V(C_{l'})$ $v'\in V(C_{l'})$ $X_vX_{v'}\geqslant 0$

$l'<l$. $v\in V(C_{l'})$ $X_v\geqslant 0$ $X_v\leqslant 0$ 1 $C_{l'}$.

$C_{l'}$ $v_1,v_2,\cdots,v_k$ i $k\geqslant 3$ $X_{v_i}\geqslant 0$ $X_{v_i}\leqslant 0$

$v_1,v_2,\cdots,v_k$ T_1 – $C_{l'}$ $C_{l''}$ $v\in V(C_{l''})$

$v'\in V(C_{l''})$ $X_vX_{v'}\geqslant 0$. $l''<l'$. $v\in V(C_{l''})$ $X_v\geqslant 0$ $X_v\leqslant 0$ 1

$C_{l'}$ l''

$l''=4$ $C_4=u_1u_2w_1w_2u_1$ $X_{u_i}\geqslant 0$ $X_{w_i}\leqslant 0(i=1,2)$. $X_{u_3}\geqslant 0$ u_1 u_2

u_3 u_1 u_2w_1 u_2u_3 u_1w_2 u_1u_3. C_4

C_3 $X_v\geqslant 0(v\in V(C_3))$ 1 $l''=2m\geqslant 6$. X_{u_1} X_{u_s} $V_{C_{2m}}^+$

$C_{2m}=u_1u_2w_1w_2\cdots u_{s-1}u_sw_{s-1}w_s\cdots w_{m-1}w_mu_1$ $u_i\geqslant 0$ $w_i<0(1\leqslant i\leqslant m)$. $s=2$ X_{u_k} $V_{C_{2m}}^+$

u_1w_m u_1u_k u_2w_1 u_2u_k C_{2m} C_3

$X_v \geqslant 0(v \in V(C_3))$ 1 $s > 2.$ u_1w_m u_1u_s C_{2m} C_{2s-2}.

$X_{u_2}X_{u_s} \leqslant X_{u_1}X_{u_{s-1}}$ u_1u_2 u_1u_{s-1} u_su_{s-1} u_su_2. C_{2s-2} C_3

$X_v \geqslant 0(v \in V(C_3))$. 1

3 4 5 6 7

$$\sum_{v_iv_j \in E(G)} X_{v_i}X_{v_j} \leqslant \sum_{v_iv_j \in E(F)} X_{v_i}X_{v_j} \leqslant \sum_{v_iv_j \in E(\bar{F})} X_{v_i}X_{v_j} \leqslant \sum_{v_iv_j \in E(\tilde{F})} X_{v_i}X_{v_j} \leqslant \sum_{v_iv_j \in E(G_1)} X_{v_i}X_{v_j} \leqslant \sum_{v_iv_j \in E(G_*)} X_{v_i}X_{v_j}$$

J I 1 n 2

$$\lambda_n(G^c) = X^TA(G^c)X = X^T(J-I)X + X^TA(G)X \geqslant X^T(J-I)X + X^TA(G_*)X = X^TA(G_*)X \geqslant \lambda_n(G_*)$$

1 G_1 G_2 $G_1(s,t)$ $G_2(s,t)$.

3 s t $s + t = n - 6(n \geqslant 12)$. $s > t$ $\lambda_n(G_1^c(s,t)) >$ $\lambda_n(G_1^c(\left\lceil \frac{n-6}{2} \right\rceil, \left\lfloor \frac{n-6}{2} \right\rfloor))$.

$\lambda_n(G_1^c(s,t)) > \lambda_n(G_1^c(\left\lceil \frac{n-6}{2} \right\rceil, \left\lfloor \frac{n-6}{2} \right\rfloor))$. $X = (X_{v_1}, X_{v_2}, \cdots, X_{v_n})^T$ $G_1^c(s,t)$

$k_1 = \lambda_n(G_1^c(s,t))$ $X_{v_i} = X_i (1 \leqslant i \leqslant n)$. $G_1^c(s,t)$ 1

$$\begin{cases} k_1X_1 = sX_3 + X_4 + X_5 + X_6 + tX_7 \\ k_1X_2 = X_5 + X_6 + tX_7 \\ k_1X_3 = 2X_1 + (s-1)X_3 + X_4 + X_5 + X_6 + tX_7 \\ k_1X_4 = 2X_1 + sX_3 + X_6 + tX_7 \\ k_1X_5 = 2X_1 + X_2 + sX_3 + tX_7 \\ k_1X_6 = 2X_1 + X_2 + sX_3 + X_4 \\ k_1X_7 = 2X_1 + X_2 + sX_3 + X_4 + X_5 + (t-1)X_7 \end{cases}$$

$(k_1I_7 - A_1)X' = 0$ $X' = (X_1, X_2, X_3, X_4, X_5, X_6, X_7)^T$

$$A_1(s,t) = \begin{bmatrix} 0 & 0 & s & 1 & 1 & 1 & t \\ 0 & 0 & 0 & 0 & 1 & 1 & t \\ 2 & 0 & s-1 & 1 & 1 & 1 & t \\ 2 & 0 & s & 0 & 0 & 1 & t \\ 2 & 1 & s & 0 & 0 & 0 & t \\ 2 & 1 & s & 1 & 0 & 0 & 0 \\ 2 & 1 & s & 1 & 1 & 0 & t-1 \end{bmatrix}$$

$f_1(x;s,t) = \det(xI_7 - A_1(s,t))$

$$\begin{aligned} f_1(x;s,t) = {} & x^7 - (t-2+s)x^6 - (6s+6t+8)x^5 - (-2st+10s+6t+22)x^4 - (-7st-10t+10)x^3 \\ & -(-4st-10s-12t-10)x^2 - (4st-3s+t-7)x - 2s - 2t \end{aligned}$$

$$f_1(x;s,t) - f_1(x;s-1,t+1) = -x(x+2)(2x-1)((s-t)(x+2)+x) \quad 8$$

$f_1(-2;s,t) = 10$ $\lambda_n(G_1^c(s,t)) < -2$. 8 $f_1(\lambda_n(G_1^c(s,t)); s-1, t+1) > 0$

$\lambda_n(G_1^c(s,t)) > \lambda_n$

$$\begin{cases} k_2X_1 = sX_3 + X_5 + X_6 + tX_7 \\ k_2X_2 = X_5 + X_6 + tX_7 \\ k_2X_3 = X_1 + (s-1)X_3 + X_4 + X_5 + X_6 + tX_7 \\ k_2X_4 = sX_3 + X_6 + tX_7 \\ k_2X_5 = X_1 + X_2 + sX_3 + tX_7 \\ k_2X_6 = X_1 + X_2 + sX_3 + X_4 \\ k_2X_7 = X_1 + X_2 + sX_3 + X_4 + X_5 + (t-1)X_7 \end{cases}$$

$(k_2 I_7 - A_2)X' = 0$ $\quad X' = (X_1, X_2, X_3, X_4, X_5, X_6, X_7)^T$

$$A_2(s,t) = \begin{bmatrix} 0 & 0 & s & 0 & 1 & 1 & t \\ 0 & 0 & 0 & 0 & 1 & 1 & t \\ 1 & 0 & s-1 & 1 & 1 & 1 & t \\ 0 & 0 & s & 0 & 0 & 1 & t \\ 1 & 1 & s & 0 & 0 & 0 & t \\ 1 & 1 & s & 1 & 0 & 0 & 0 \\ 1 & 1 & s & 1 & 1 & 0 & t-1 \end{bmatrix}$$

$f_2(x;s,t) = \det(xI_7 - A_2(s,t))$

$$f_2(x;s,t) = x^7 - (t-2+s)x^6 - (5s+5t+4)x^5 - (-2st+5s+3t+10)x^4$$
$$-(-5st-5s-6t+3)x^3 - (st-6t-3t-4)x^2 - (3st+s+2t-2)x + st - s$$

$$f_2(x;s,t) - f_2(x;s-1,t+1) = -(x+1)(2x-1)((s-t)x^2 + (2s-2t-2)x - s + t) \qquad 9$$

$f_2(-3;s-1,t+1) = -344 + 114t + 2s + 28(s-1)(t+1) > 0$ $\quad \lambda_n(G_2^c(s-1\ \ t+1)) < -3.$

9 $\quad f_2(\lambda_n(G_2^c(s-1,t+1));s,t) < 0.$ $\quad \lambda_n(G_2^c(s,t)) > \lambda_n(G_2^c(s-1,t+1)).$

2

2 $\quad$ 3 $\quad$ 4

1 $\quad G^c \in C_n^c (n \geqslant 12)$

$$\lambda_n(G^c) \geqslant \max\{\lambda_n(G_1^c(\lceil\frac{n-6}{2}\rceil, \lfloor\frac{n-6}{2}\rfloor)), \lambda_n(G_2^c(\lceil\frac{n-5}{2}\rceil, \lfloor\frac{n-5}{2}\rfloor))\}$$

$$G \cong G_1(\lceil\frac{n-6}{2}\rceil, \lfloor\frac{n-6}{2}\rfloor) \quad G \cong G_2(\lceil\frac{n-5}{2}\rceil, \lfloor\frac{n-5}{2}\rfloor).$$

1 $\quad \lambda_n(G_1^c(22,22)) < \lambda_n(G_2^c(23,22)).$

$n = 50$ $\quad f_1(x;22,22) = x^7 - 42x^6 - 272x^5 + 594x^4 + 3598x^3 + 2430x^2 - 1885x - 88$ $\quad f_2(x;23,22) = x^7 - 43x^6 - 229x^5 + 821x^4 + 2774x^3 - 298x^2 - 1583x + 483.$ $\quad x < 4.2$ $\quad f_1(x;22,22) - f_2(x;23,22) > 0$.

$f_2(-4.2;23,22) = 92025.63258 > 0$ $\quad \lambda_n(G_1^c(22,22)) < \lambda_n(G_2^c(23,22)).$

2 $\quad \lambda_n(G_1^c(201,200)) > \lambda_n(G_2^c(201,201)).$

$n = 407$ $\quad f_1(x;201,200) = x^7 - 399x^6 - 2414x^5 + 77168x^4 + 283390x^3 + 165220x^{22} - 1.60$

2 HONG Y SHU J L. Sharp Lower Bounds of the Least Eigenvalue of Planar Graphs J . Linear Algebra and Its Applications 1999 296 01-03 227-232.

3 LIU Z ZHOU B. On Least Eigenvalues of Bicyclic Graphs with Fixed Number of Pendant Vertices J . Journal of Mathematical Sciences 2012 182 02 175-192.

4 WANG Y FAN Y Z. The Least Eigenvalue of Graphs with Cut Edges J . Graphs and Combinatorics 2012 28 04 555-561.

5 LI S WANG S. The Least Eigenvalue of the Signless Laplacian of the Complements of Trees J . Linear Algebra and Its Applications 2012 436 07 2398-2405.

6 YU G D FAN Y Z YE M L. The Least Signless Laplacian Eigenvalue of the Complements of Unicyclic Graphs J . Applied Mathematics and Computation 2017 306 03 13-21.

7 JIANG G YU G SUN W et al. The Least Eigenvalue of Graphs Whose Complements Have Only Two Pendent Vertices J . Applied Mathematics and Computation 2018 331 05 112-119.

8 FAN Y Z ZHANG F F WANG Y. The Least Eigenvalue of the Complements of Trees J . Linear Algebra and Its Applications 2011 435 09 2150-2155.

9 WANG Y FAN Y Z LI X X et al. The Least Eigenvalue of Graphs Whose Complements are Unicyclic J . Discussiones Mathematicae Graph Theory 2015 35 02 249-260.

The Graft Transformations and Their Applications on the Least Eigenvalues of Graphs

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Abstract $V(G)=\{v_1,v_2,\cdots,v_n\}$ G $A(G)=(a_{ij})n\times n$ $a_{ij}=1$ v_i v_j $a_{ij}=0$ $\lambda_1(G)\geqslant\lambda_2(G)\geqslant\cdots\geqslant\lambda_n(G)$ $n\geqslant 12$

Keywords